Exercise 1
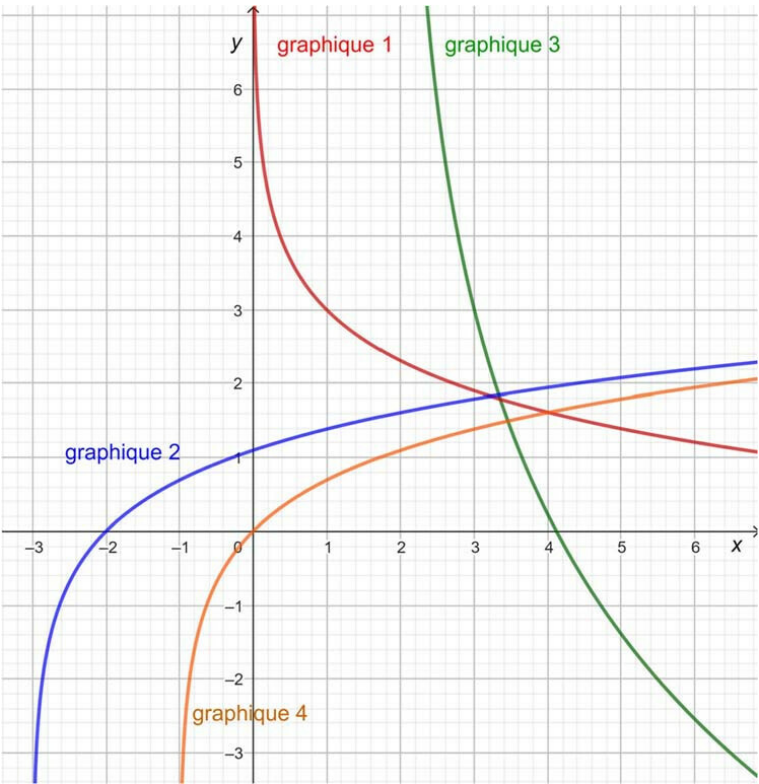
We give three logarithmic functions f, g and h defined respectively by

$$f(x) = 3 - 4\ln(x-2), \qquad g(x) = 3 - \ln(x) \qquad \text{et} \qquad h(x) = \ln(x+3)$$

as well as four graphs of logarithmic functions depicted below.

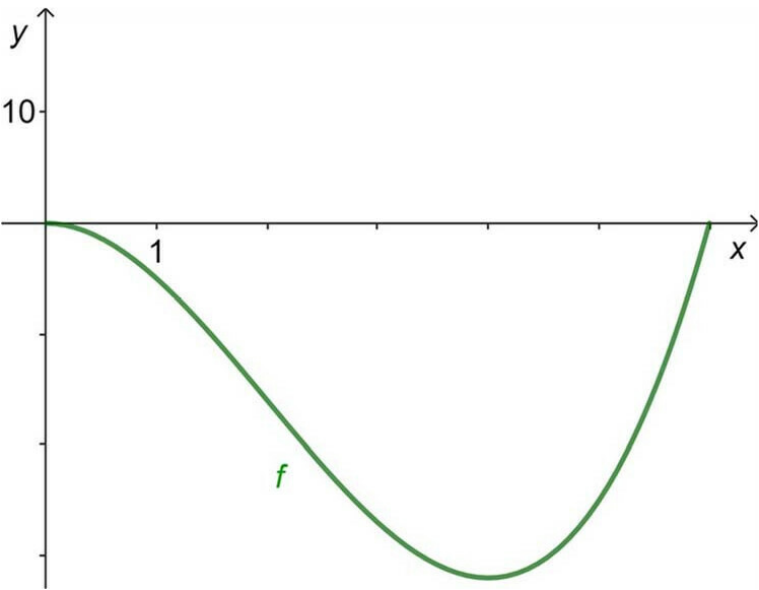**Associate** each of the three functions with the corresponding graph. **Justify** the answer. | 5 marks

Exercise 2

The shape of the bottom of a lake can be modeled by the function f defined by

$$f(x) = x^3 - 6x^2, \qquad 0 \le x \le 6$$

where x is measured in kilometers and $f(x)$ in meters.

Determine the depth of the lake at its deepest point. | 5 marks

Exercise 3	Calc. : ✗

We know that the function F defined by $F(x) = x^2 + 2x$ is a primitive of the function f and that $\displaystyle\int_1^a f(x)\,dx = 5$, where a is a positive real number.
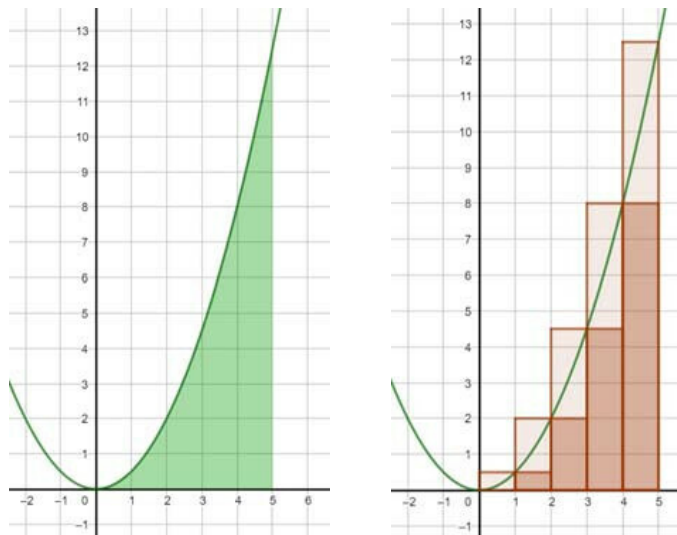
Determine a. | 5 marks

Exercise 4 | Calc. : ✗

We consider the function f defined by $f(x) = \dfrac{x^2}{2}$.

We seek to determine the area A of the surface delimited by the graph of f, the x-axis and the lines of equations $x = 0$ and $x = 5$. (see diagram below left)

a) Using the rectangles represented (see diagram above at right), **determine** a lower and an upper bound of the area A sought. | 2 marks

b) **Explain** how to obtain finer bounds with this technique. | 1 mark

c) **Show** that the function F defined by $F(x) = \dfrac{x^3}{6}$ is an antiderivative of the function f and calculate the exact value of the area A. | 2 marks

---

Exercise 5 | Calc. : ✗

The Ferris wheel of an amusement park has a diameter of 50 meters. It makes a complete turn uniformly every 120 seconds. Its highest point is located 55 meters from the ground. We consider the movement of a basket of the Ferris wheel. This is a periodic movement which can be modeled by a function f defined by

$$f(t) = a \cdot \sin(b(t - c)) + d$$

where t represents the time, in seconds, and $f(t)$ the height of the gondola, in meters.
The basket is at the lowest point of the Ferris wheel at the moment $t = 0$.

a) **Show** that the amplitude of the movement is equal to 25 meters. | 1 mark

b) **Show** that the vertical displacement is equal to 30 meters. | 1 mark

c) **Show** that $b = \dfrac{\pi}{60}$. | 1 mark

d) **Determine** c and **interpret** the result. | 2 marks

Exercise 6 | Calc. : ✗

During a 100 m race, athlete Ali is placed in lane 3 on the starting line.
There are 8 corridors in total.
Three other athletes taking part in the race are placed in the other corridors.
Calculate the probability that none of the other three runners are placed at Ali's side. | 5 marks

Exercise 7   Calc. : ✗

Lactase is an enzyme that helps digest dairy products. containing lactose.
30% of the world's population produces its own lactase in intestines.
A medical company has developed a new rapid test to see whether or not an individual produces lactase. A study is carried out to determine the reliability of the rapid test and the study shows that this test gives a correct positive result for 80% of individuals who produce lactase but gives a false positive for 10% of individuals who do not produce lactase.
The rapid test is used on a large population.

   a) **Show** that 69% of the population should get a negative result.   3 marks

   b) **Calculate** the probability that an individual produces lactase, knowing that he obtained a   2 marks
      negative result.

Exercise 8   Calc. : ✗

Andy is a basketball player. The probability that he succeeds a free throw is 75%.
He is allowed 3 attempts and each successful throw is worth one point.

   a) Does the number of points scored follow a binomial law? **Justify** the answer.   3 marks

   b) **Calculate** the probability that Andy scores at least one point.   2 marks

Exercise 9   Calc. : ✗

A company produces chocolate bars whose mass follows a normal law with mean $\mu = 100$ g and standard deviation $\sigma = 1$ g.
We choose a chocolate bar at random from the production.

   a) **Determine** the probability that this tablet weighs between 97 g and 103 g.   3 marks

   b) **Determine** the probability that this tablet weighs more than 100 g.   2 marks

Exercise 10   Calc. : ✗

After some complaints about the new schedules, management of one school claims that only 10% of teachers are unhappy with their new schedule. Some teachers think that it is more than 10%. They then ask their opinion to a group of 35 teachers chosen at random.
An NHST test is carried out at a significance level of 5%.

   a) **Determine** if this test is one-sided left or right. **Justify** the answer.   1 mark

   b) **Formulate** an appropriate null hypothesis H_0 and an alternative hypothesis H_1 for this test.   1 mark

The random variable X designates the number of teachers unhappy with their new schedule in a sample of 35 teachers.
The table below shows the values of $P(X \geq k)$ with $k = 1, 2, 3, 4, 5\ 6, 7, 8, 9$ and 10 assuming that 10% of teachers are unhappy.

k	1	2	3	4	5	6	7	8	9	10
$P(X \geq k)$	0.975	0.878	0.694	0.469	0.269	0.132	0.055	0.020	0.006	0.002

   c) **Determine** the critical value k and **interpret** this value.   3 marks