Exercise 1 Calc. : ✓

In this question, parts 1, 2 and 3 are independent.

Part 1.

Sports watches are wristwatches that can be used during sporting activities. A lot of people use those watches.
The so-called Sporty sports watch is particularly popular. The probability that a random person with a sports watch has the watch Sporty is 60%.
We are looking at a sample of 500 people with sports watches. The random variable X gives the number of people in this sample that have the sports watch Sporty.

a) **Explain** why X can be modelled by a binomial law and **give** its parameters. — 2 marks

b) **Calculate** the probability that at least 300 people in this sample have the sports watch Sporty. **Round** to 2 decimal places. — 2 marks

c) **Determine** the expected number of people in this sample with the sports watch Sporty. — 2 marks

d) **Calculate** the standard deviation of X. **Round** to 3 decimal places. **Interpret** it in the given context. — 2 marks

Part 2.

The sports watch Sporty can give the effort during a run very accurately if the person gives his or her weight.
A woman with a weight of 60 kg is running uphill for 30 minutes. Therefore, her effort level is not steady. Her running power can be modelled by the following function:

$$P(t) = -0.05t^2 + 3t + 66, \qquad \text{with } 0 \leq t \leq 30$$

where t is in minutes and $P(t)$ in kJ/min (kilojoules per minute).

e) **Calculate** at which power the woman is running when she starts running, and 15 minutes after she started. — 3 marks

f) **Draw** the graph of the function P in the given domain. — 3 marks

g) **Determine** at what time the woman's running power is 106 kJ/min. — 3 marks

Part 3.

A lot of people are using the internet to buy their sports watch Sporty, and ask for a delivery at a shop called "RunAway".
We know that 80% of the time the Sporty arrives on time (in a few days), 15% of the time it arrives late (it takes some weeks to arrive) and the rest of the times it doesnt arrive at all.
We also know that when the Sporty arrives on time, the probability that people like the shop "RunAway" is 0.9; when it arrives late, the probability that people like it is 0.3; and if it doesnt arrive at all the probability that people like the shop is 0.1.
We randomly select a user who ordered a Sporty watch online and asked for delivery in this shop.

h) **Sketch** a tree diagram of the situation above. — 3 marks

i) **Compute** the probability that this user likes the shop "RunAway". — 2 marks

j) If we know that this person liked the shop, **give** the probability that the Sporty that was ordered arrived on time. — 3 marks

Exercise 2 Calc. : ✓

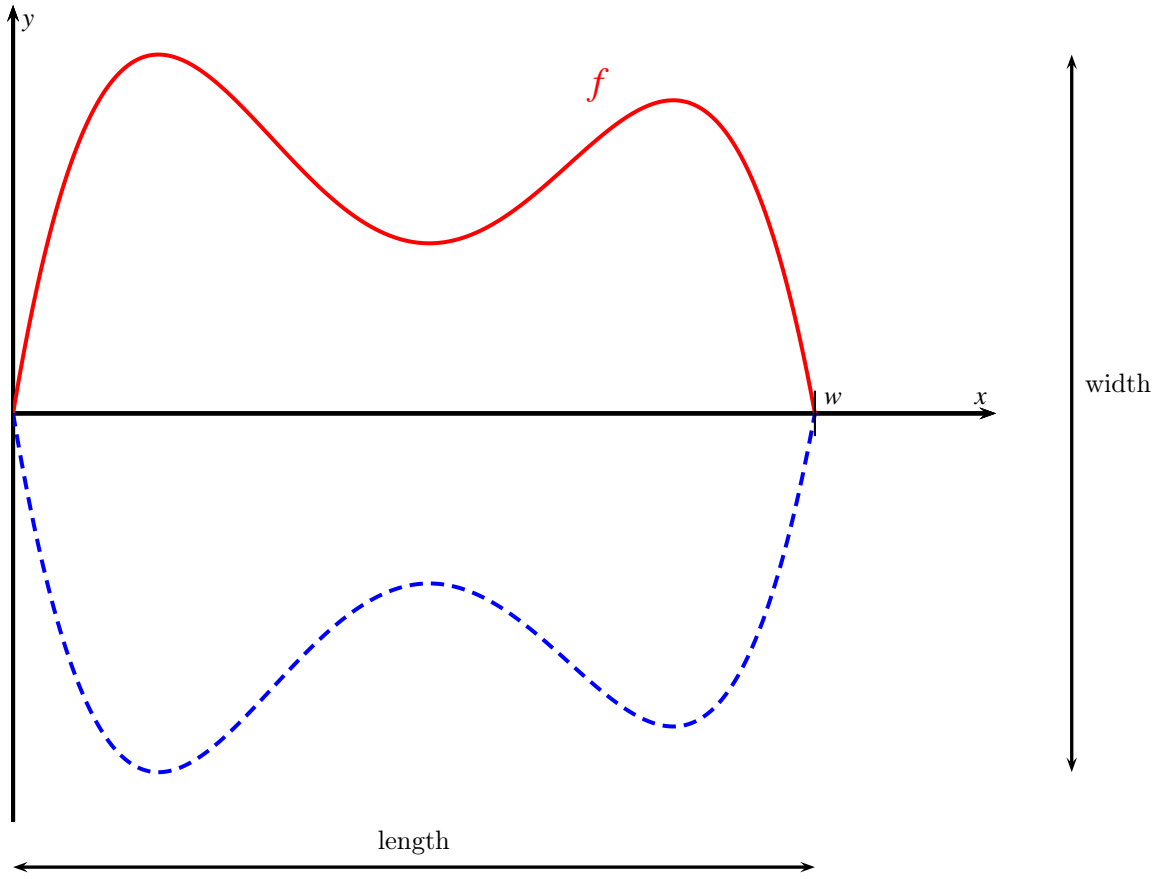
In this question, parts 1 and 2 are independent.

Part 1.

A musician plays a guitar and wishes to model its shape. The main wood box can be modeled by the following equation:

$$f(x) = -0.13x^4 + 1.4x^3 - 4.9x^2 + 6x$$

The following picture shows the curve of f (in red, plain line), together with the symmetric of this curve, with respect to the x-axis (in blue, dashed line). In this equation, x is in decimetres, and $f(x)$ is also in decimetres. The surface between those two curves forms the wood box of this guitar.



As can be seen on the graph, the function f is in fact defined from 0 to a value w, which is the other solution of the equation $f(x) = 0$.

a) **Determine** the value of w, **rounding** to 3 decimal places. **Give** the length of the wood box, in centimetres.	2 marks
b) **Determine** the maximum value of f, **rounding** to 3 decimal places. **Give** the width of the wood box, in centimetres.	2 marks
c) The function f has three stationary points. In question b) we have found one of them. **Give** the coordinates of the two other stationary points, **rounded** to two decimal places.	4 marks
Before a big concert, our musician wants to paint the back of the wood box in black. We hence want to know what is the area of this surface.	
d) **Determine** an approximate value of the following integral, **rounded** to 3 decimal places: $$\int_0^{5.3} f(x)\,\mathrm{d}x$$ **Give** the area that has to be painted, in square decimetres.	3 marks

Part 2.

Our musician opens a webpage for his band, and is interested in the number of followers across time ($x = 0$ when the webpage is created). The table below shows the number of followers for the first 20 weeks:

x = Time (weeks)	2	4	5	8	10	11	12	13	16	18
y = Number of followers	275	240	180	300	380	350	250	350	440	400

e) **Draw** a scatter diagram to represent the data from the table. — 3 marks

f) **Compute** the linear correlation coefficient. **Determine** if a linear model would be appropriate for his data. **Discuss** how we could improve the linear model by combining it with another one. — 3 marks

g) **Determine** an equation in the form $y = a \cdot x + b$ of the linear regression of y on x using this data. **Round** a and b to one decimal place. — 3 marks

Draw the regression line on the same diagram as e).

In h) and i), use the linear model $f(x) = 20 \cdot x + 190$.

h) **Compute** when the number of followers would be over 800. — 3 marks

i) **Explain** why the model is not appropriate over many weeks. — 2 marks