Exercise 1 Calc. : ✓

The high-speed train (TGV) going to Gare Centrale in Luxembourg City starts slowing down when it passes the train station of the town of Bettembourg. The train velocity, v, in m/s, is given by:

$$v(t) = 84 - 0.3t$$

...where t is the time in seconds after the train has passed the Bettembourg train station.
For the following questions, you may use the formulae:

- The distance d (in meters) covered by an object moving at a velocity $v(t)$ between time a and time b is given by: $d = \int_a^b |v(t)|\,\mathrm{d}t$.
- The acceleration a (in m/s^2) of an object moving at a velocity $v(t)$ is given by: $a = \frac{\mathrm{d}v(t)}{\mathrm{d}t}$. It can be positive or negative.
- The thermal energy E (in J, Joules) generated between time a and time b by the TGV train moving at a velocity $v(t)$ is given by: $E = 220\,000 \int_a^b v(t)\,\mathrm{d}t$.

a) **Find** the distance the train has covered in 100 seconds after it has gone through the Bettembourg train station. — 2 marks

b) **Calculate** the acceleration of the train (here a deceleration). — 2 marks

c) **Justify** with an appropriate calculation that the train stops 280 seconds after it has gone through the Bettembourg train station, keeping in mind that the train is considered to have stopped when its velocity is equal to zero. — 3 marks

d) During deceleration, there is heat buildup in the brakes of the train due to friction. **Calculate** the thermal energy generated in the train brakes between the start of the deceleration process in Bettembourg and the point at which the train comes to a complete stop in Gare Centrale. — 2 marks

The TGV generates noise. The government is investigating using trees and bushes to mitigate the effects of this noise. Through dense foliage, the train noise attenuation A, measured in decibels per meter, dB/m, is given by the following model:

$$A(x) = -0.058 + 0.0177 \cdot \ln(x)$$

...where x is the propagation distance of the noise, measured in meters from the train.

e) **Calculate** the attenuation right past the bushes, 30 meters away from the train. — 3 marks

f) **Find** the minimal distance away from the train that provides an attenuation of at least 0.04 dB/m. — 3 marks

g) **Discuss** the limit of the attenuation value when the propagation distance approaches infinity. — 2 marks

People reach the train station on time with a probability of 89%. On a winter's day, 210 people want to ride a train. Let X be the number of persons who reach their train on time. We assume that X follows a binomial distribution.

h) **Determine** the probability that none of the persons reach their train with a delay. — 2 marks

i) **Find** the probability that at least 200 of the persons reach their train on time. — 2 marks

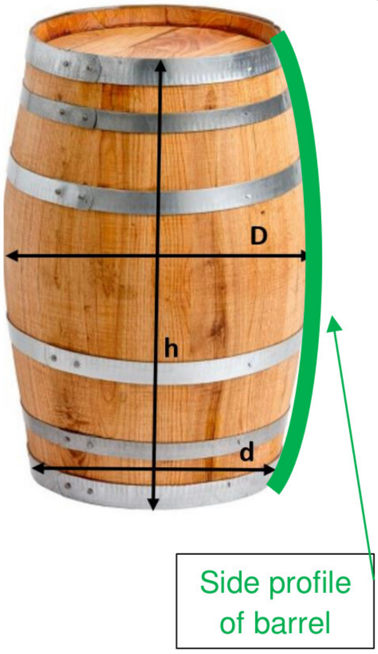
j) **Find** the probability that less than 90% of this group will reach their train on time. — 2 marks

k) **Calculate** the expected value and the standard deviation of X. — 2 marks

Exercise 2 Calc. : ✓

A Luxembourgish winery produces wines that are partially fermented in oak barrels.
On average, the barrel used has the dimensions shown in the figure opposite:

- Height: h=10 dm
- Minimum diameter: d=5 dm
- Maximum diameter: D=6 dm

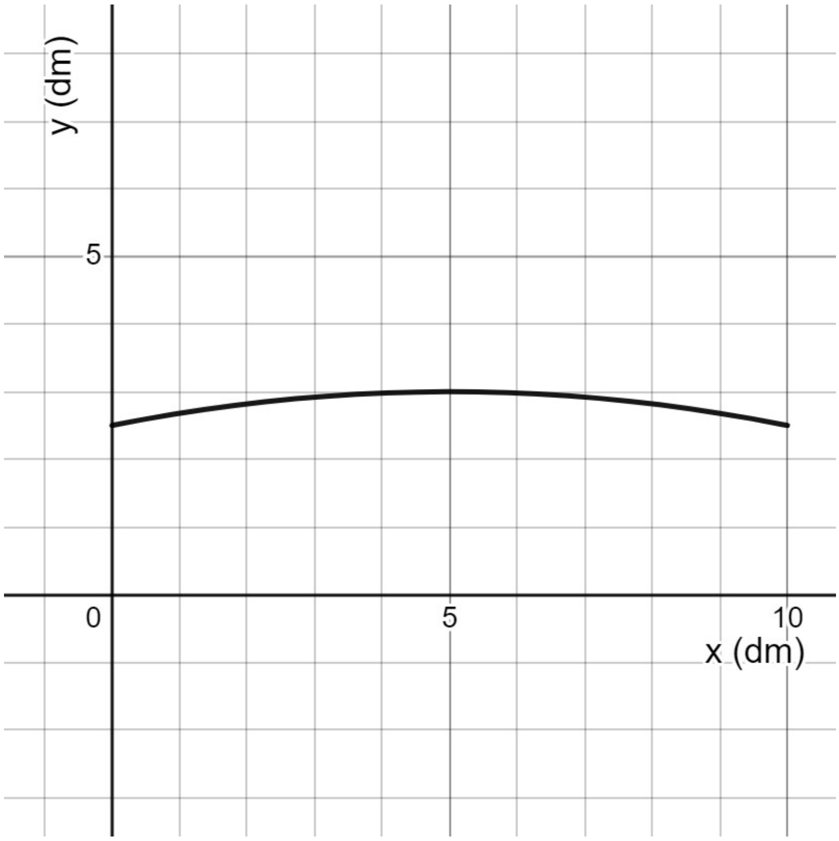


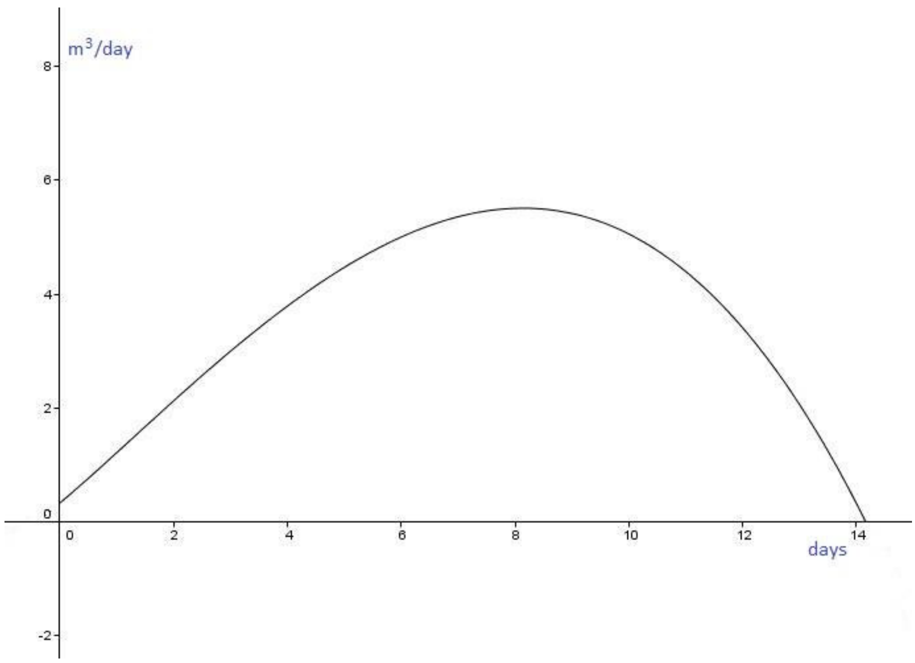
The side profile of the barrel can be modelled with a function

$$f(x) = -0.02x^2 + 0.2x + 2.5$$

...where the radius of the barrel $f(x)$ is represented as a function of the height x (with $0 \leq x \leq 10$ and the measurements are expressed in decimeters).

The side profile is also shown in the following figure:



a) **Calculate** $f'(5)$ and **verify** that the barrel has a maximum diameter of 6 dm.	2 marks
b) Knowing that the formula for the length of a curve is $$l = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$ ... **calculate** the length of the side profile in dm (express the result to an accuracy of three decimal places)	2 marks
c) According to the barrel manufacturer, the capacity of the average barrel is between 252 and 253 dm^3. **Verify** that the company's statement is correct using the formula for the volume of a rotating solid: $$V = \pi \int_a^b (f(x))^2\, dx$$	2 marks
d) After fermentation, the wine is transferred to barrels with a flow in cubic meters per day given by: $$g(t) = -0.005 \cdot t^3 + t + \frac{2}{3 \cdot t + 6}$$ where t is the time after the transfer starts, measured in days. The graph of function g is represented here: 	
i. **Calculate** the volume of wine transferred between day 2 and day 14 (inclusive).	2 marks
ii. **Determine** when the rate of wine transfer is at a maximum.	2 marks

Part of the wine produced is used to make a liquor wine for aging. As the years pass, the wine evaporates. The following table shows the amount of wine left $w(t)$ in a barrel filled in the year 1990, with respect to time t (in years), with t starting from $t = 1990$.	

t (year)	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
Wine left (liters)	252	252	251	251	249	247	244	244	243	240	239

Enter the data into your calculator.

e) Interpreting the trend observed in the data for liquid remaining as a function of year, **explain** why the data appear to be correlated.	1 mark
f) Assuming a linear model can be applied, **determine** the equation of the linear regression line. Give the coefficients of the equation with a precision of two decimal places.	2 marks
g) **Determine** the linear regression coefficient with a precision of two decimal places. **Interpret** the result; is the correlation reliable?	1 mark
h) For this question, we will take: $w(t) = -1.418t + 3\ 076$. **Estimate** how much wine will be left in a barrel after 20 years (your answer should be correct to the nearest liter).	2 marks

The wine is finally sold in bottles. The promotional campaign for the product requires that the inside of the bottle cap contains a code that gives the buyer a chance to win a prize with a probability $p = 0.093$. In a shopping center, 100 bottles are simultaneously displayed.

i) **Justify** the fact that one can use a binomial distribution with probability p to model this situation.	1 mark
j) **Calculate** the mean value and variance of the binomial distribution.	2 marks
k) **Calculate** the probability (with a precision of 4 decimal places) that there are at least 2 bottles out of 100 with a winning cap.	2 marks
l) **Calculate** the probability (with a precision of 4 decimal places) that there are exactly 5 bottles out of 100 with a winning cap.	2 marks
m) **Calculate** the probability (with a precision of 4 decimal places) that there are a maximum of 10 bottles out of 100 with a winning cap.	2 marks