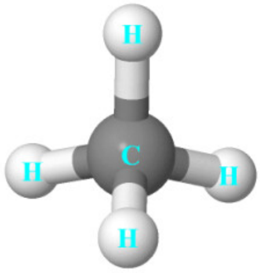
Exercise 1 | Calc. : ✓

The methane molecule CH_4, represented opposite, can be modeled by a regular tetrahedron OABD, with $O(0;0;0)$, $A\left(3;\ \sqrt{3};2\sqrt{6}\right)$, $B\left(3;3\sqrt{3};0\right)$ and $D(6;0;0)$.
The four vertices are the positions of the hydrogen atoms H.
We are interested in this exercise in the position of the carbon atom C, inside this tetrahedron. This position is represented by a point that we call G.



1. The tetrahedron is regular, so all of its edges have the same length. Justify that this length is equal to 6. — 1 mark
2. Justify that the coordinates of the orthogonal project A′ of A on the plane (Oxy) with equation $z = 0$ are $A'\left(3;\ \sqrt{3};0\right)$. — 2 marks
3. The point G is the intersection of (AA′) and (IJ), where $I\left(\frac{3}{2};\ \frac{\sqrt{3}}{2};\ \sqrt{6}\right)$ is the middle of segment [AO] and $J\left(\frac{9}{2};\ \frac{3\sqrt{3}}{2};0\right)$ the middle of the segment [BD].
   (a) Prove that the coordinates of G are $\left(3;\ \sqrt{3};\ \frac{\sqrt{6}}{2}\right)$. — 3 marks
   (b) Check that the distance between G and A is equal to $\dfrac{3\sqrt{6}}{2}$. — 1 mark
   We admit that it is also the distance between G and the other vertices of the tetrahedron.
   (c) In reality, the length of the C–H bond is approximately equal to 109 picometers. Determine an approximate value, in picometers, the distance between two hydrogen atoms. — 2 marks
4. Give an approximate value of the measure of the angle formed by two C–H bonds. — 3 marks

Exercise 2 | Calc. : ✓

1. Let the complex number $w = \dfrac{1}{4} + \dfrac{1}{4}\mathrm{i}$.
   (a) Write w in exponential form. — 2 marks
   (b) Determine the values of the natural number n for which w^n is a real number. — 3 marks
2. For any natural number n, we note M_n the affix point z_n defined by:

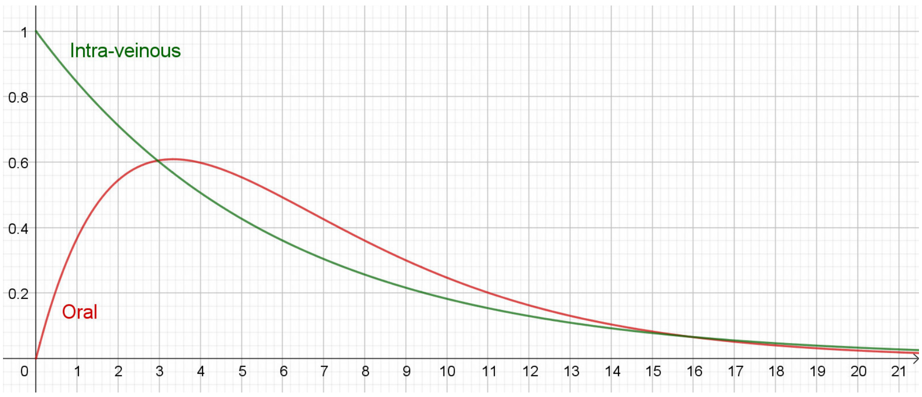
$$\begin{cases} z_0 = 1 \\ z_{n+1} = \left(\dfrac{1}{4} + \dfrac{1}{4}\mathrm{i}\right) \cdot z_n, \quad n \in \mathbb{N} \end{cases}$$

   (a) Calculate z_1 and z_2, then place in the complex plane the points M_0, M_1, M_2 (graphical unit: 4 cm). — 2 marks
   (b) Let r be the sequence defined, for any natural number n, by $r_n = |z_n|$. — 3 marks
   Show that the sequence r is geometric with commonality $\dfrac{\sqrt{2}}{4}$.
   Deduce an expression of r_n as a function of n.
   (c) We assume that for any natural number n, $z_n = r_n e^{\frac{in\pi}{4}}$. — 1 mark
   Under what condition does the point M_n belong to the real axis?
   (d) Describe the precise position of the point M_{10} which represents z_{10} in the complex plane. — 2 marks

Exercise 3 Calc. : ✓

The graphs below show the concentration of a drug in a patient's blood stream, with two different types of one-time injection — intra-veinous and oral — as a function of the time t, in minutes, over the interval $[0, 20]$.

The value 1 on the y-axis denotes the initial concentration at the time of the intra-veinous injection.



Use the graphs to answer questions **1.** to **4**. Give approximate values with the precision allowed by the graphs.

1. Describe the variations of concentration after an intra-veinous injection. — 1 mark
2. At what time does the oral injection reach its maximum concentration? What is then the value of that concentration? — 1 mark
3. Give approximate values of the coordinates of the inflection point after an oral injection. What does it mean for the rate of change in the concentration at that moment? — 2 marks
4. Over what interval of time it the concentration higher after an oral injection than it is after an intra-veinous injection? — 2 marks

Functions f and g that model these concentrations are defined by:

$$f(t) = \mathrm{e}^{-0.17t} \quad \text{and} \quad g(t) = t \cdot \mathrm{e}^{-0.3t-0.7}$$

5. Explain the way in which function f models the intra-veinous injection and function g models oral injection. — 3 marks
6. Use the calculator to find the times when the two concentrations are equal. Give approximate values to the thousandth. — 2 marks
7. The area under the curve (AUC) of one of these functions gives the total exposure to the drug over a certain period. Compute that value over the first five minutes, for the two types of injections. — 2 marks

   Give a detailed answer, with all the steps in the computation.

Exercise 4 Calc. : ✓

<u>In this exercise, all results will be rounded to 3 decimal places.</u>

1. Here is the evolution of sales of electrically assisted bicycles in France between 2007 and 2017.

Year	2007	2009	2011	2013	2015	2017
Rank of the year x_i	0	2	4	6	8	10
Number of e-bikes sold (thousands): n_i	10	23	37	57	102	278

<u>Data: Observatoire du Cycle</u>

   (a) Using the calculator, determine an affine adjustment of n in x for this data. Specify the correlation coefficient. — 2 marks

   (b) We set $y_i = \ln(n_i)$. In this case, a affine adjustment of y in x is given by the formula $y = 0.307x + 2.353$, with a correlation coefficient approximately equal to 0.981. — 2 marks

   Use best-fit model to extrapolate bike sales of this type in 2023.

2. A company mass-produces power-assisted bicycles. Either X the random variable which, for each bike taken at random in the production associates its autonomy, in kilometres. We admit that this random variable X follows a normal law.

   We know that $P(X \geq 84) = 0.2266$ and $P(X \leq 86) = 0.8943$.

   Determine the mean and the standard deviation of this law. Round the results in km to the nearest integer. — 4 marks

3. In this part, it is considered that 4% of lithium-ion batteries have a defect and are described as "non-compliant".

   Let Y be the random variable which, for any batch of 150 batteries taken randomly in production, associates the number of batteries not compliant.

   The production is large enough for us to be able to assimilate such a collection of 150 batteries in a draw with replacement.

   (a) Determine $P(Y \leq 5)$ by specifying the chosen model. — 2 marks

   (b) Determine the probability that, in a random sample of 150 batteries, all batteries are compliant. Interpret the result. — 2 marks