Exercise 1 Calc. : ✗

Let f be the function defined on $(0, +\infty)$ by $f(x) = a + b\dfrac{\ln(x)}{x}$.
The representative curve of the function f admits an asymptote horizontal with equation $y = 1$ and a tangent at the point of abscissa 1 with equation $y = -x + 2$.
Determine the values of a and b.

5 marks

Exercise 2 Calc. : ✗

In a three-dimensional space, we consider:

- The line L_1 of parametric representation: $\begin{cases} x = 3 + \lambda \\ y = -3 - 4\lambda \\ z = -2 + 2\lambda \end{cases} \quad (\lambda \in \mathbb{R})$
- The point $A(2, 1, -4) \in L_1$
- The line L_2 of parametric representation: $\begin{cases} x = 10 - 3\mu \\ y = -21 + 12\mu \\ z = 11 - 6\mu \end{cases} \quad (\mu \in \mathbb{R})$

Show that L_1 and L_2 are parallel then determine the coordinates of point B of line L_2 such that the line (AB) is perpendicular to L_1 and L_2.

5 marks

Exercise 3 Calc. : ✗

Solve in $\mathbb{R}$ the equation $16^{x^2} = 2^{4x-1}$.

5 marks

Exercise 4 Calc. : ✗

Calculate the integral:

5 marks

$$\int_{-1}^{1} \frac{3}{2}\left(e^{3x} + e^{-3x}\right) dx.$$

Exercise 5 Calc. : ✗

A metal chain hangs between two walls.
Its height above the ground level can be described by the equation:

$$h(x) = e^{-x} + e^{x-1} + 2,$$

where x is the distance in meters along the ground from the left wall.
Calculate how many meters from the left wall this chain is closest to the ground.

5 marks

Exercise 6 Calc. : ✗

In the complex plane, show that the set of points M with affix z checking equality:

5 marks

$$|z - 1 - 3i| = |z + 2 - 3i|$$

is a straight line for which we give an equation.

Exercise 7 Calc. : ✗

An electronic device makes it possible to obtain randomly in whole natural x included, in the broad sense, between 1 and 999 (we are therefore in a situation of equiprobability). Any number between 10 and 99 is written with two digits and any number between 1 and 9 is written with a single digit ; thus the number sixty-two will be displayed 62 and not 062, likewise the number seven will be written 7 and not 007.

1. Show that the probability of getting a multiple of 5 is $\frac{199}{999}$. — 3 marks
2. Calculate the probability that the same number appears at least twice times in writing x. — 3 marks
3. In this question we will round the probability of obtaining a multiple from 5 to 0.2.
   5 numbers are successively determined using this device.
   Calculate the probability that, among these five numbers, three exactly be multiples of five. — 3 marks
4. We model the choice of a **real** number x in the interval $[1;999]$ by a random variable following the density law defined by the function $f(x) = \frac{1}{998}$.
   (a) What is the probability of rolling a multiple of 5? — 1 mark
   (b) What is the probability of getting a real less than or equal to 500? — 3 marks

Exercise 8 Calc. : ✗

Let a be a non-negative real number.
We consider the equation

$$(E) : \ln(x) = ax^2.$$

Study the number of solutions of this equation according to the value of a. — 7 marks