Exercise 1 Calc. : ✓

Part 1

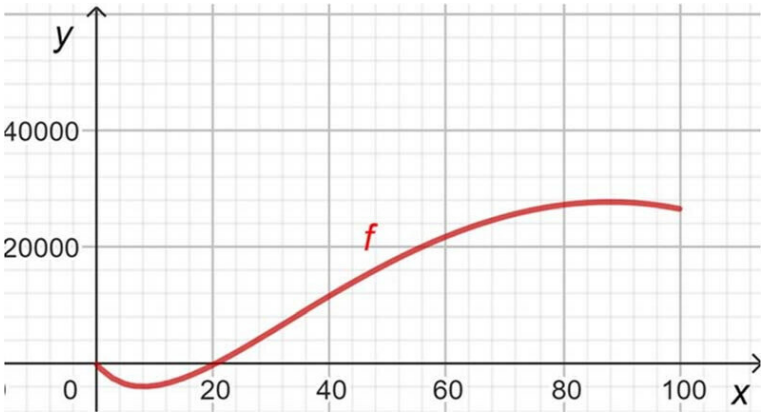
Apples from an orchard are used to produce apple juice. Given the size of the orchard, the quantity of juice produced per year may not exceed 100 tons.

The net result (profit or deficit), in euros, achieved by the farmer is modeled by

$$f(x) = 2160x - 10x^2 - 40\,000 \ln\left(\frac{x+12}{12}\right)$$

where x is the number of tons of apple juice.

The diagram below shows the graph of the function f.



a) **Calculate** the profit made by the sale of 50 tons of apple juice. — 1 mark

b) **Determine** the number of tons of apple juice that the farmer must produce to obtain a profit (positive net result). Round to a whole number of tonnes. — 3 marks

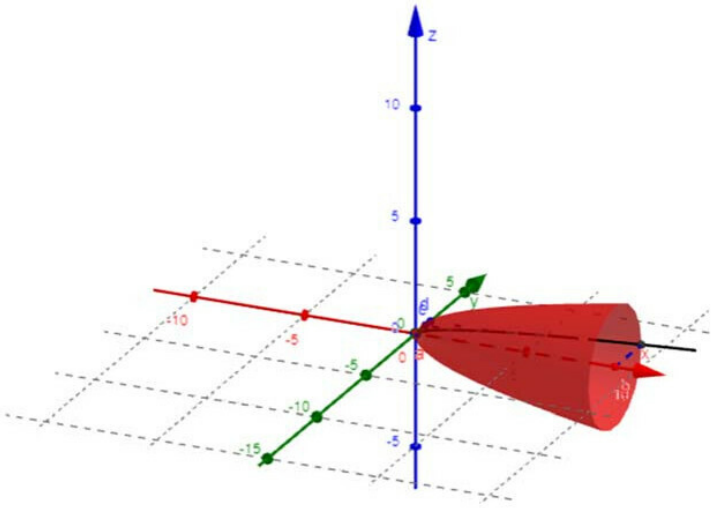
c) The derivative of the function f is given by:

$$f'(x) = 2160 - 20x - \frac{40\,000}{x+12}$$

Use the derivative to **determine** the number of tons of apple juice to produce to obtain the maximum benefit. **Calculate** this maximum profit. — 4 marks

Part 2

The farmer wants to organize a tasting.
For this event, he plans to use glasses whose shape is modeled by the function f defined by $f(x) = \sqrt{0.8x}$ of which we rotate the graph around the x-axis for $0 \leq x \leq 9$.
The result is shown in the figure below.



d) To calculate the volume of such a solid of revolution, we use the formula:

$$V(x) = \pi \cdot \int_a^b (f(x))^2 \, \mathrm{d}x$$

If x is measured in cm, the volume $V(x)$ will be given in cm^3.

Calculate the volume of a glass and give the answer in liters. Round to two decimals. — 2 marks

e) We also need to set up tables for the tasting. The company that provides the glasses offers them in 6 different colors. We also have 3 different sizes of napkins.

Determine how many ways you can set a table using glasses of 2 different colors with napkins of 2 different sizes. — 2 marks

f) To advertise the event, the farmer wants to use the following slogan:

"*Eating apples makes you happy*".

A survey was carried out, the results of which are grouped in the table next, where

X is the number of apples consumed per week and

Y is the assessment of personal happiness on a scale from 1 to 10

X	4	9	5	2	1	0	9	7	5
Y	5	4	9	6	4	5	8	4	2

.

Determine the Pearson correlation coefficient and **justify** if the slogan is correct or not, based on statistics. — 3 marks

Part 3

The farmer uses different kinds of apples from his orchard to produce apple salads:

60% *Elstar* apples  and 40% *Boskoop* apples. 

We admit that for this fruit salad, 97% of apples are *Elstar* and 95% of *Boskoop* apples are of good quality.

g) The farmer takes an apple at random to check it.

Show that the probability that the apple is of poor quality is 0.038. — 3 marks

The farmer packages his apples in boxes of 60.

We note Y as the random variable which designates the number of poor quality apples per box.

h) **Justify** that Y follows a binomial distribution. — 3 marks

i) We choose a box at random.

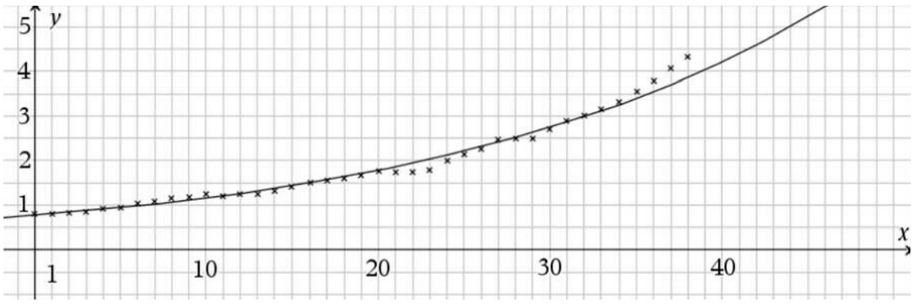
Calculate the probability that there are exactly 2 poor quality apples in the box. Round to two decimal places. — 2 marks

j) **Calculate** $E(Y)$ and **explain** the meaning of the result. — 2 marks

Exercise 2 Calc. : ✓

Part 1

The diagram below shows the evolution of the number of passengers in global air transport between 1980 and 2018, as well as the graph of an exponential function f which models this evolution.
The number x of years after 1980 corresponds to the number $f(x)$ of passengers in billions.



a) Is the given exponential model relevant? **Justify** the answer. — 2 marks

b) We consider the following definitions of the model f:

$$f_1(x) = 0.75 \cdot e^{0.043x} \quad ; \quad f_2(x) = e^{0.043x} \quad ; \quad f_3(x) = 0.75 \cdot e^{-0.043x}$$

Designate the definition that best corresponds to the graph represented above and **justify** the answer. — 3 marks

c) We consider the model $f(x) = e^{0.0431x-0.284}$

Calculate the number of passengers that this model can predict in 2023. Does this model still make sense in 2023? **Justify** the answer. — 3 marks

Part 2

It is estimated that the probability that an air passenger will not show up for the takeoff is 0.05. An airline that sells seats for a 100-seat plane decides to sell 103 seats to "overbook" and thus hopes to make additional profits if some passengers do not show up at departure.

d) **Calculate** the probability that at least one passenger arriving at departure will not find a seat on the plane. Round to 5 decimal places. — 3 marks

e) Tickets for this plane are sold at 200 each. If a passenger shows up and does not have a seat on the plane, the company owes them 800 of compensation.

We then obtain the following table, where X designates the number of passengers arriving for departure, and Y the impact of "overbooking", in euros, on the result of ticket sales, depending on of X.

X	≤ 100	101	102	103
Y	+600	−200	−1000	−1800
Probability	0.8935	0.0739	0.0275	0.0051

.

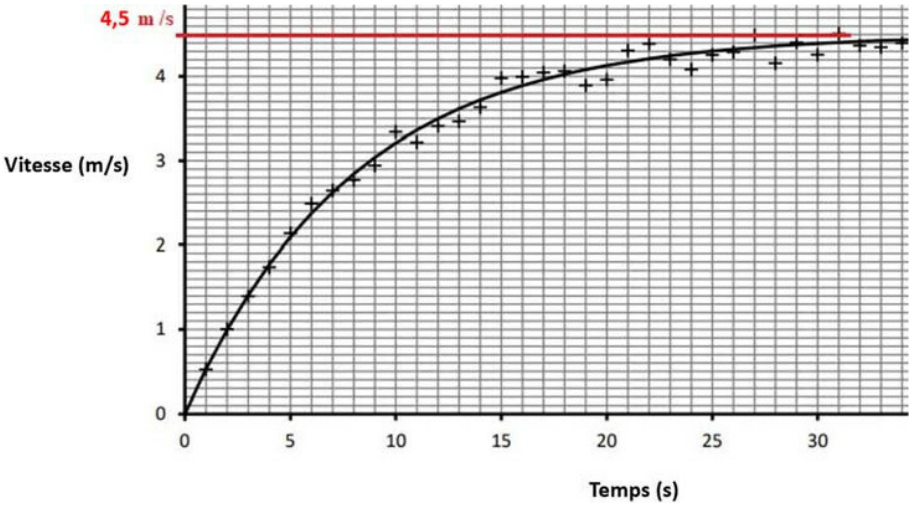
Calculate $E(Y)$ and **interpret** the result. Is it advantageous for this company to "overbook"? — 4 marks

Part 3

In order to reduce noise pollution and CO2 emissions, the movement of an aircraft on the ground is done without using its main engines (reactors), but electric motors. The plane, initially stationary, starts on level ground; electric engine power does not allow it to exceed a maximum speed $v_{\max}$. The speed of the aircraft, expressed in meters per second (m/s), is modeled by the function v defined by

$v(t) = 4.5 \cdot \left(1 - e^{0.13t}\right)$, where $t \geq 0$ is the time expressed in seconds (s).

The diagram below shows the graph of this function; the crosses represent the experimental values.



f) **Determine** the limit of the function v at plus infinity and **interpret** the result. 3 marks

g) **Determine** the initial acceleration of the plane. 3 marks

h) **Calculate** $\int_{10}^{20} v(t)\,\mathrm{d}t$ and **interpret** the result. 4 marks