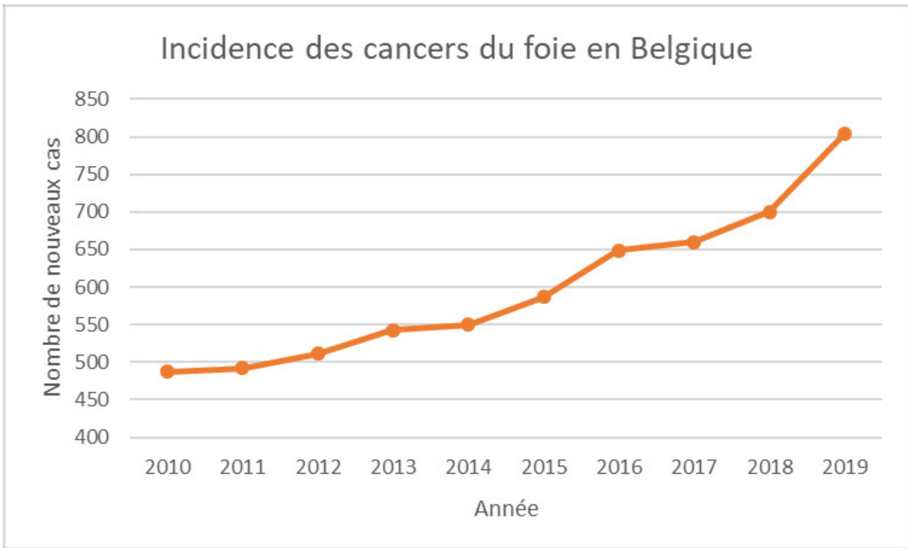
**Exercise 1** Calc. : ✓

| | |
|---|---|
| Following the introduction of 100 squirrels into a forest in 2020, the population is studied. It is observed that the number of squirrels increases by an average of 30% each year.<br>It is assumed that the squirrel population can be modeled by a function of the form:<br>$$P(x) = k \times A^x$$<br>where $\begin{cases} P \text{ is the number of squirrels} \\ x \text{ is the time in years} \\ k \text{ and } A \text{ are constants to be determined} \end{cases}$ | |
| a) **Determine** the value of the constant $A$ of the model corresponding to the data given. **Justify** your response. | 2 marks |
| For the rest of the exercise, we use the following function:<br>$$P(x) = 100 \times 1.3^x$$ | |
| b) **Calculate** the squirrel population after 6 months; 5 years; 10 years. | 3 marks |
| c) Using the calculator, **determine** the year in which the squirrel population will exceed 500 squirrels. | 2 marks |
| d) **Explain** why this model cannot be used in the long term. | 2 marks |
| In order to study the squirrel population in 2021, a feeder was installed in the middle of the forest. A camera is placed nearby, as well as a heat detector with a tally counter. Every hour of the day, the number of squirrels present at the feeder is counted.<br>During a day in 2021, a maximum number of visitors (10 squirrels) was detected at 8:00 in the morning. At 20:00, the minimum number of visitors was counted (no squirrels).<br>It is assumed that the use of the feeder over time can be modelled by a periodic function. | |

| | |
|---|---|
| e) **Determine** the parameters $a$, $b$, $c$ and $d$ in the model of the type: $$N(x) = a \cdot \sin(b \cdot (x - c)) + d$$ where $N$ is the number of squirrels present at the feeder and $x$ is the hours in a day. | 3 marks |
| In 2020, a similar study was carried out. The number of squirrels at the feeder over time was modeled in this earlier study by the following function: $$T(x) = 6 \cdot \sin\left(\frac{\pi}{10} \cdot (x - 3)\right) + 7$$ | |
| f) Using the periodic model for the year 2020, **calculate** the number of squirrels at 14:00. | 2 marks |
| g) **Represent** the periodic pattern. | 2 marks |
| h) Using the periodic model for the year 2020, **estimate** the hour(s) of the day when there are 7 visitors to the feeder. | 3 marks |
| It is estimated that 10% of the squirrels in the forest are chipped in order to be traced. | |
| i) **Calculate** the probability of having at least 1 chipped squirrel among the squirrels present at the feeder at 08:00 in the morning. Round the answer to 4 decimal places. | 3 marks |
| When the squirrel population reaches 1 000 individuals, an epidemiological study is conducted. It is observed that 60% of squirrels are females, out of which 15% are still of the generation introduced in 2020; while 250 squirrels are males of one of the following generations.<br>We define the following events:<br>• **F** = the squirrel is a female<br>• **M** = the squirrel is a male<br>• $G_0$ = the squirrel is of the generation initially introduced<br>• $G_1$ = the squirrel is of one of the generations following the one initially introduced | |
| j) A squirrel from the starting generation is taken at random. **Determine** the probability that this squirrel is a male. | 3 marks |

**Exercise 2** Calc. : ✓

Medical data collected in recent years by the Cancer Registry show that the number of new cases (incidences) of liver cancer continues to increase in Belgium.




Two models are proposed to model this evolution:

$$f(t) = 32.7818t + 450.78$$

$$g(t) = 463.93 \times 1.0123^t$$

where $t$ is the time in years from 2010.

a) Using the calculator, **determine** for each of the two models in which year Belgium will reach 1 000 new cases of liver cancer per year. — 2 marks

At the end of 2021, 803 new cases of liver cancer were detected.

b) **Calculate**, using each of the two models, the projected number of new cases of liver cancer in Belgium in 2021. **Compare** the results obtained with the number actually observed and **deduce** the most appropriate model for the situation. — 4 marks

c) **Calculate** $\int_0^9 f(t)\,\mathrm{d}t$ and **interpret** the result obtained in the context described in the statement. — 2 marks

Studies show that several risk factors may be involved in the occurrence of liver cancer. Among these factors is an infection with the hepatitis C virus.
In Belgium, 1% of the population is infected with the hepatitis C virus.
There is a test for this virus with the following properties:

- The probability of an infected person having a positive test is 0.98 (sensitivity of the test).
- The probability of an uninfected person having a negative test is 0.97 (specificity of the test).

A randomly selected person from the Belgian population is tested at random.
We define the following events :

- V the event "the person is infected with hepatitis C virus"
- T the event "the test is positive".

d) **Write** this information with correct mathematical probability notations. — 3 marks

e) **Represent** the given situation using a probability tree. — 3 marks

f) **Demonstrate** that the probability of the test being positive is 0.0395. — 2 marks

g) A person has just received the result of his test: negative. **Determine** the probability that this person is indeed not infected with the virus. Round the answer to 4 decimal places. — 3 marks

20 people from the population are chosen at random. The “draws” are considered independent. The random variable $X$ gives the number of people infected with the hepatitis C virus among these 20 people.

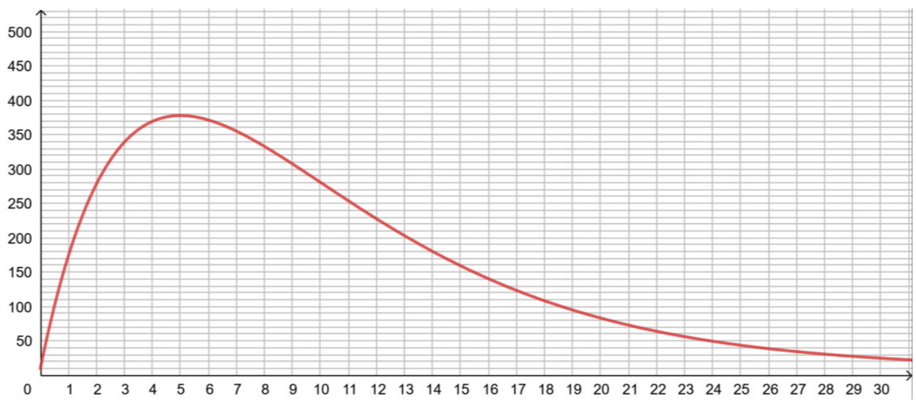
h) **Calculate** the probability that there are at least two infected people among the 20. Round this result to 4 decimal places. — 3 marks

---

An infection with the Hepatitis C virus results in a change in the number of white blood cells (immune cells involved in the body's defence mechanisms) in the blood that irrigates the liver.
The white blood cell count was measured in a patient within 30 days of the infection with the hepatitis C virus.
The evolution of the white blood cell count can be modeled by the following function, shown in the graph below :

$$f(x) = 200x \cdot e^{-\frac{x}{5}} + 10$$

White blood cell count ($\times 10^9$ cells per liter of blood)




Time (days)

i) Using this model, **determine** the rate of change in the white blood cell count 3 days after the infection. — 3 marks