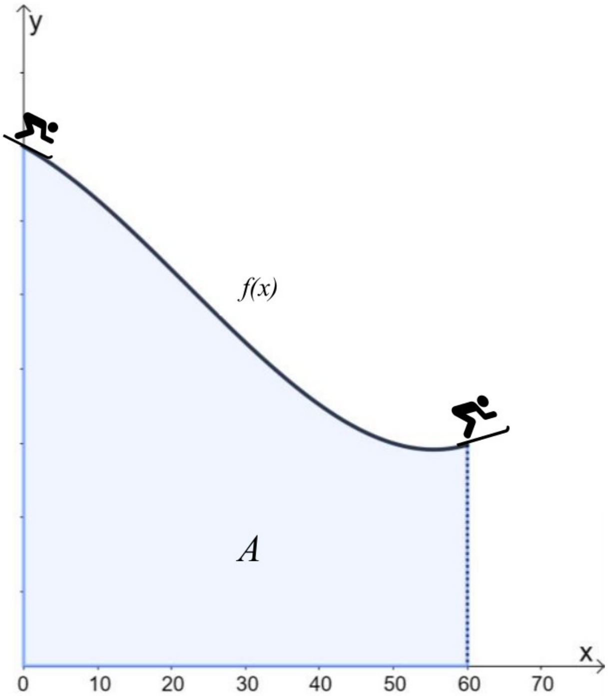
Exercise 1 Calc. : ✓

Ski Jump

Part 1 (Parts 1, 2 and 3 of this question can be solved independently.)

The ramp of a ski jump is shown in the diagram below and can be modelled by the function $f(x)$.



The function $f(x)$ is defined in the interval shown in the diagram with the equation:

$$f(x) = \frac{3}{10\,000}x^3 - \frac{1}{50}x^2 - \frac{11}{20}x + 70$$

, where $f(x)$ and x are expressed in meters.

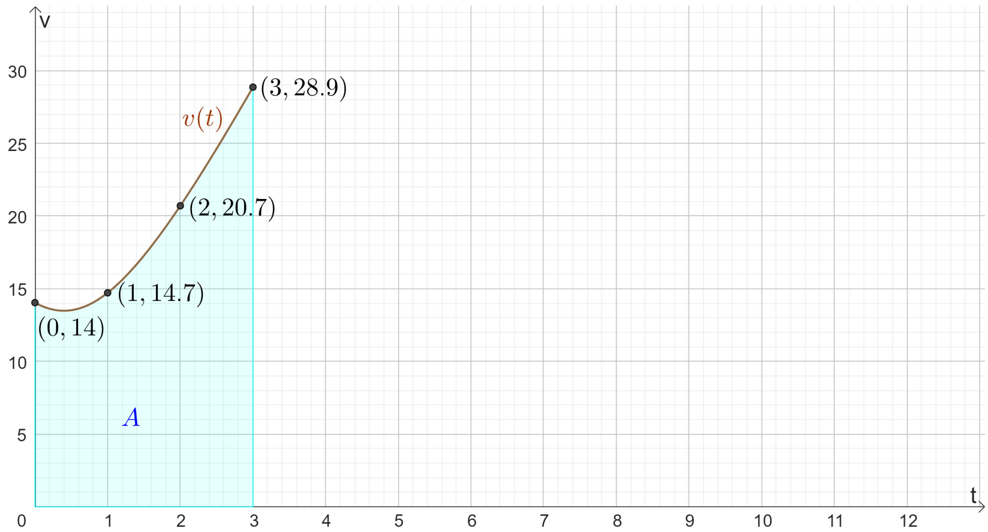
a) Use the equation and the information in the graph to **determine** the domain of $f(x)$.	2 marks
b) **Calculate** the area A.	3 marks
c) When a skier is at the end of the ramp, the skis define a tangent line r to the graph of $f(x)$. **Define** this tangent line and show every step in your calculation.	4 marks
d) The skier is at the lowest point on the ski ramp. **Calculate** the height at the lowest point on the ski ramp. **Explain** your method.	4 marks

Part 2

Use the following definitions for Parts 2 and 3:

- The position of an object is determined by the function $s(t)$, where t is the time in seconds and $s(t)$ is expressed in meters.
- The velocity function $v(t)$ is defined as $v(t) = s'(t)$.
- The acceleration function $a(t)$ is defined as $a(t) = v'(t)$.

After taking off from the ramp, the skier flies through the air until he lands on the ground. The time between take-off and landing is exactly 3 seconds. The velocity function $v(t)$ (in m/s) of the flying skier is shown in the graph below (with t in seconds).



e) **Find** the velocity (in m/s) of the skier when he lands on the ground.	1 mark
f) Use the available information in the diagram to **calculate** an approximation for the area A. **Explain** your method.	3 marks
g) Is the approximation for the area A from question f) an underestimation or an overestimation of the exact area? **Justify** your answer.	2 marks
h) **Interpret** what the exact area A means in the given context.	2 marks

Part 3 As the skier lands on the landing slope, he slows down until he comes to a complete stop. The velocity of the skier on the landing slope can be modelled by the function: $$v(t) = -3.4 \cdot t + 28.9$$ where t is in seconds and $t = 0$ corresponds to the moment when the skis touch the ground.	
i) How long does it take for the skier to slow down to a complete stop? **Justify** your answer.	2 marks
j) **Investigate** whether a landing slope of 120 m is long enough for the skier.	2 marks
	

Exercise 2 Calc. : ✓

The Island

Part 1 (Parts 1 and 2 of this question can be solved independently.)

The table below gives the measured population on an island.

Beginning of the year	2015	2020
Population	5 500	7 250

a) Use a linear model to **predict** the population at the beginning of 2023. 2 marks

b) Peter uses an exponential model $p(t) = k \cdot a^t$ to model the population. In this model, $t = 0$ corresponds to the beginning of 2015 and a and k are parameters.

Find the parameters a and k of the model $p(t)$. 3 marks

c) **Show** that the exponential model $f(t) = 5\ 500 \cdot e^{0.05525 \cdot t}$ adequately fits the given data. 2 marks

For questions d), e) and f), you can use the exponential model

$$f(t) = 5\ 500 \cdot e^{0.05525 \cdot t}$$

In this model $t = 0$ corresponds to the beginning of 2015.

d) **Determine** the annual growth rate of the exponential model. 2 marks

e) **Calculate** $f'(5)$ and **interpret** what the result means in the given context. 2 marks

f) Use the exponential model to **find** in which year the population would reach 10 000 people. 3 marks

At the beginning of 2022, the island was hit by an earthquake. Although nobody was hurt in the event, 6 000 people decided to leave the island immediately. After they left, the growth rate of the island population was the same as before.

g) **Investigate** in which year the island population will be the same as it was at the beginning of 2015. 3 marks

Part 2

The day length is the time between sunrise and sunset. Peter lives on the island and measured the day length of every first day of the month during a whole (non-leap) year. The results are given below:

Date	1^{st} of Jan	1^{st} of Feb	1^{st} of Mar	1^{st} of Apr	1^{st} of May	1^{st} of Jun
Daylength (in hours)	7.67	8.55	10	11.2	12.33	13

Date	1^{st} of Jul	1^{st} of Aug	1^{st} of Sep	1^{st} of Oct	1^{st} of Nov	1^{st} of Dec
Daylength (in hours)	13.05	12.67	11.6	10.35	8.95	7.83

Peter models the day length $h(x)$ with the periodic model $h(x) = a \cdot \sin(b(x - c)) + d$, where $h(x)$ is expressed in hours, x is expressed in days and $x = 1$ corresponds to the 1^{st} of January.

h) **Explain** why the day length $h(x)$ can be modelled with a periodic model and **give** the period of this model. 2 marks

i) **Estimate** the amplitude of this periodic model. 2 marks

j) Hence, **investigate** for which values of the parameters a, b, c, and d the periodic model $h(x)$ fits the data adequately. 4 marks