



## MATHEMATICS 3 PERIODS

### PART A

DATE : 23<sup>rd</sup> of January 2023, afternoon

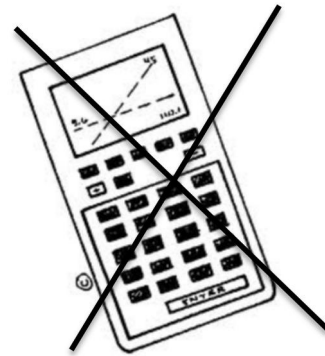
#### DURATION OF THE EXAMINATION:

2 hours (120 minutes)

#### AUTHORIZED MATERIAL:

Examination without technological tool:

Pencil for the graphs



#### SPECIFIC INSTRUCTIONS:

This exam paper consists of 8 printed pages, including the cover page (1/8 to 8/8), with an annex page. This annex page may be detached from the paper but must be handed in with the booklet.

Read the questions carefully and show all your workings clearly.

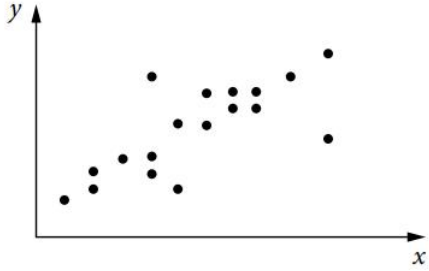
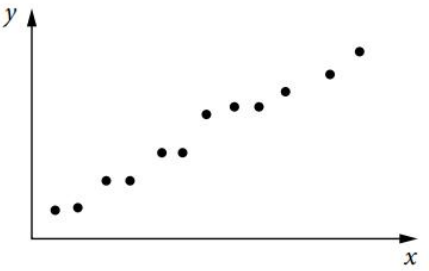
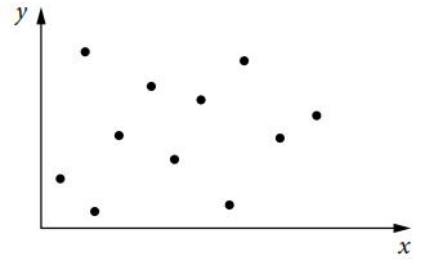
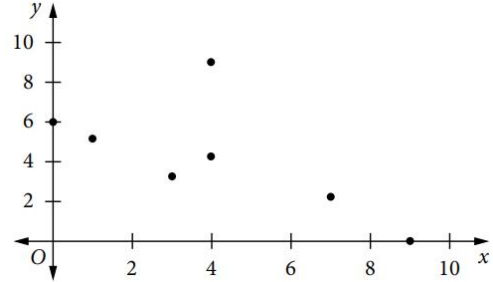
If graphs are used to find a solution, they must be sketched as part of the answer.

Any attempt of cheating will result in the cancellation of your examination on the spot.

Last Name	First name
Mark ..... / 50	Final grade

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**PART A**

<b>Questions S1 to S10 are independent</b>		<b>Marks</b>
S1	<p>Answer the following multiple choice questions. No justification is needed.</p> <p>There is one good answer per question.</p> <p>One mark is awarded per correct answer. No mark penalty for wrong answers.</p>	5 marks
	<p>a) Which statement characterises the data shown on the scatter diagram?</p> <ol style="list-style-type: none"> <li>1. Weak, positive, linear trend</li> <li>2. Moderate, positive, linear trend</li> <li>3. Moderate, negative, linear trend</li> <li>4. Strong, negative, linear trend</li> </ol>	
	<p>b) For the scatter diagram shown, what is the value of <math>r</math> ?</p> <ol style="list-style-type: none"> <li>1. <math>-1 &lt; r &lt; -0.7</math></li> <li>2. <math>-0.5 &lt; r &lt; -0.3</math></li> <li>3. <math>0.3 &lt; r &lt; 0.5</math></li> <li>4. <math>0.7 &lt; r &lt; 1</math></li> </ol>	
	<p>c) For the scatter diagram shown, what is the value of <math>r</math> ?</p> <ol style="list-style-type: none"> <li>1. <math>-1 &lt; r &lt; -0.7</math></li> <li>2. <math>-0.5 &lt; r &lt; -0.3</math></li> <li>3. <math>-0.2 &lt; r &lt; 0.2</math></li> <li>4. <math>0.3 &lt; r &lt; 0.5</math></li> </ol>	
	<p>d) For the scatter diagram shown, the Pearson's coefficient <math>r</math> was found to be <math>-0.6</math> .</p> <p>The point <math>(4, 9)</math> was found to be recorded incorrectly and should have been plotted as <math>(4, 1)</math>.</p> <p>Based on this change, what is the correct coefficient <math>r</math> ?</p> <ol style="list-style-type: none"> <li>1. Positive but closer to 0</li> <li>2. Positive but closer to 1</li> <li>3. Negative but closer to 0</li> <li>4. Negative but closer to <math>-1</math></li> </ol>	

e) A scatter diagram is shown with its line of best fit.

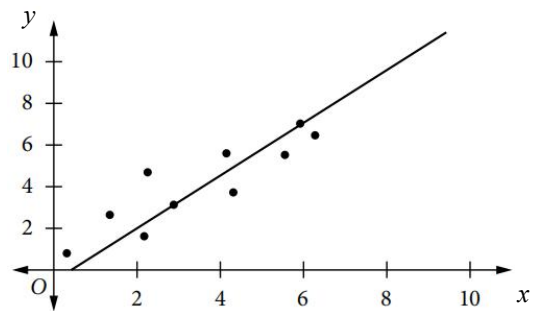
What is the equation of the line of best fit ?

1.  $y = 4x - 3$

2.  $y = \frac{4}{3}x + 1$

3.  $y = \frac{4}{3}x - 1$

4.  $y = \frac{3}{4}x + 1$



S2 Over eight consecutive years, a city nursery has measured the growth of an outdoor bamboo species for that year. The annual rainfall in the area where the bamboo was growing was also recorded. The data are shown in the table below.

Rainfall (mm)	450	620	560	830	680	650	720	540
Growth (cm)	25	45	25	85	50	55	50	20

The scatter diagram of the above data is shown on the annex page .

(to be handed in)

a) Given the mean point is approximately (630,44), **draw** the line of best fit by eye on the diagram. 2 marks

b) Use this line to **estimate** the growth for a rainfall reading of 500 mm. 1 mark

c) Use this line to **estimate** the rainfall for a given year if the growth was 30 cm. 1 mark

d) **Explain** why your answers in b) and c) are reliable. 1 mark

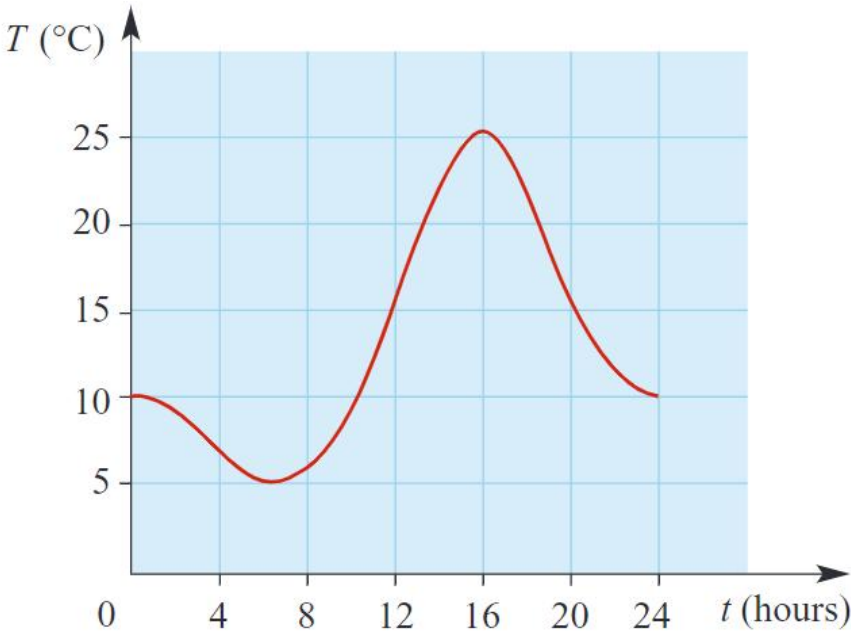
S3 a) Let  $f$  be the function defined on  $[1; 10]$  by  $f(x) = x^2 - 12x + 96$ .

**Find** the variations and extremum of  $f$  and **display** the results in a table of variations. 3 marks

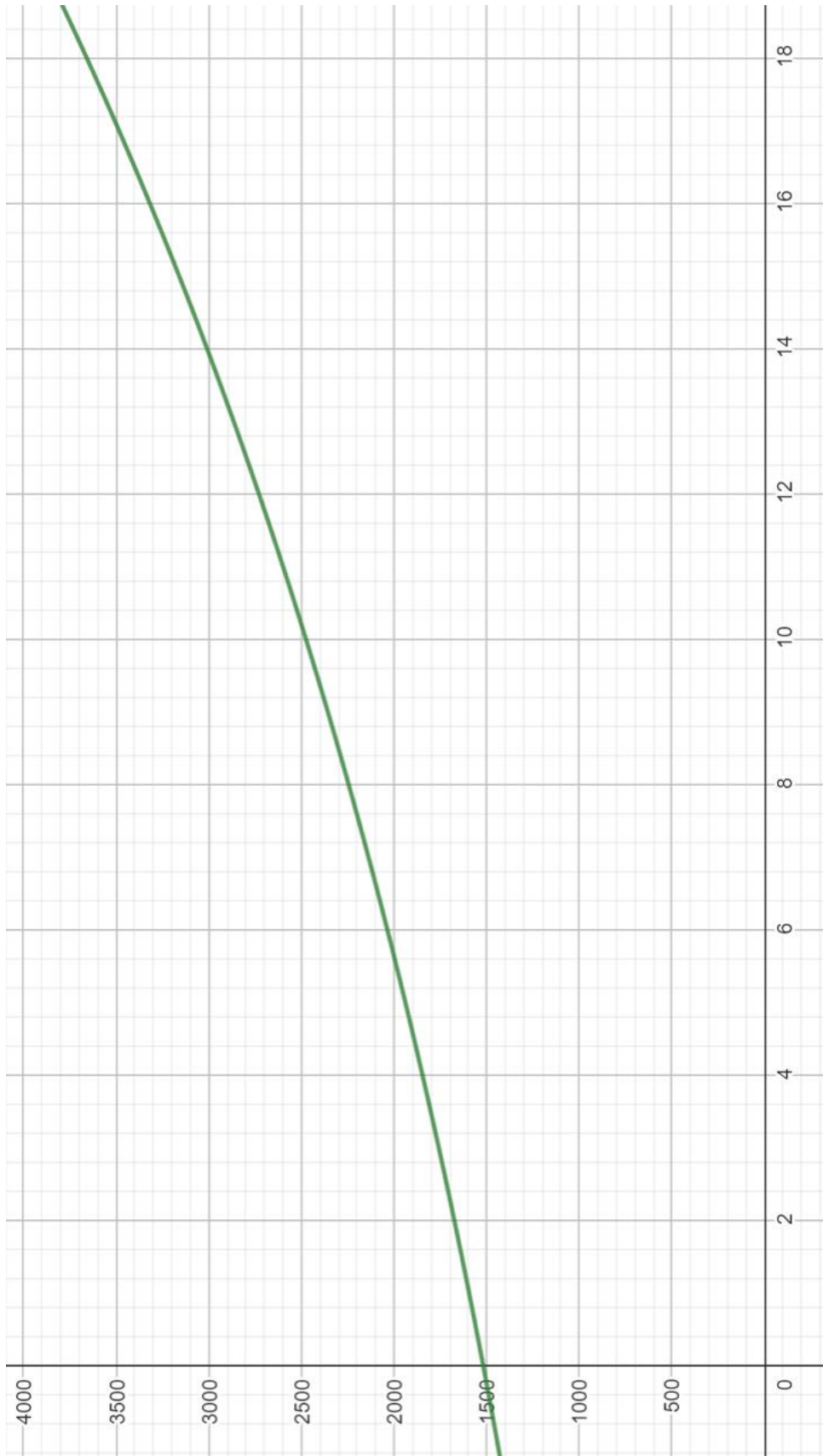
b) A small computer factory can produce up to 10 computers per week. We denote  $x$  the number of computers produced per week. We admit that for any whole number from the interval  $[1; 10]$ , the total producing cost is equal to  $f(x)$ , expressed in tens of euros.

**Find** the number of computers that should be produced in a week so that the cost would be minimal and **give** the value of that cost. 2 marks

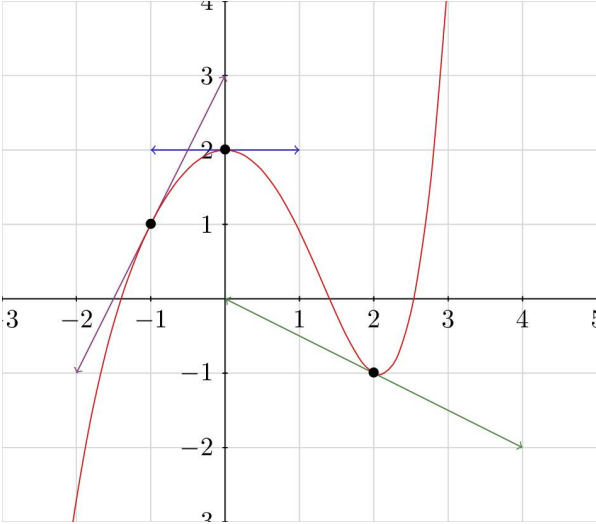
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<p>S4</p>	<p>Temperature (<math>T^{\circ}\text{C}</math>) varies with time (<math>t</math> hours) over a 24-hour period, as illustrated in the graph.</p>  <p>a) <b>Estimate</b> the maximum temperature and the time at which this occurs.</p> <p>For questions b) and c) give your answer to the nearest half <math>^{\circ}\text{C}</math> per hour.</p> <p>b) The temperature rise between 10:00 and 14:00 is approximately linear. <b>Estimate</b> the average rate at which the temperature is increasing in this period.</p> <p>c) <b>Estimate</b> the instantaneous rate of change at <math>t = 20</math>.</p>	<p>1 mark</p> <p>2 marks</p> <p>2 marks</p>
<p>S5</p>	<p>The monthly number of visitors in a museum is modelled by <math>v(t) = 1520 \times (1.05)^t</math> where <math>t</math> is the number of months since the opening of the museum in may 2020.</p> <p>a) <b>Interpret</b> the numbers 1520 and 1.05 in this context.</p> <p>The graph of <math>v</math> given on the next page will be used to answer questions b) and c).</p> <p>b) <b>Estimate</b> the number of visitors in December 2020.</p> <p>c) The museum will need to hire an additional member of staff if the number of visitors gets greater than 3000. <b>Determine</b> the date of recruitment of this agent.</p>	<p>2 marks</p> <p>1 mark</p> <p>2 marks</p>

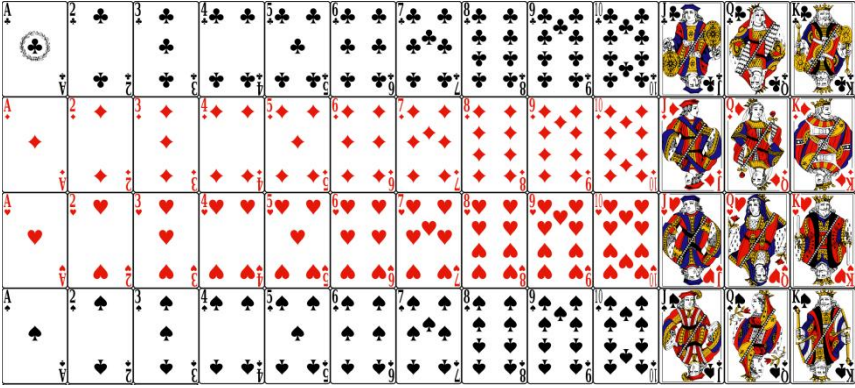
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<p>S6</p>	<p>Below is shown the graph of a function <math>f</math> and three of its tangents.</p>  <p><b>Indicate</b> if the statement is true or false. No justification is needed. (one mark per statement).</p> <p>a) <math>f'(0) = 0</math></p> <p>b) <math>f'(2) = -2</math></p> <p>c) <math>f(x) \geq 0</math> on the interval <math>[2; 3]</math></p> <p>d) <math>f'(x) \leq 0</math> on the interval <math>[0; 2]</math></p> <p>e) The equation <math>f(x) = 2</math> has two solutions on the interval <math>[-2; 4]</math></p>	<p>5 marks</p>
<p>S7</p>	<p>32 students are asked if they can play the piano and/or guitar. Answers are:</p> <p>15 play the piano</p> <p>8 play the piano and guitar</p> <p>21 play at least one of the two instruments</p> <p>a) <b>Construct</b> a Venn diagram to display the information and calculate all the possible numerical values that could be displayed on the diagram.</p> <p>b) A student is chosen at random, <b>calculate</b> the probability that this student plays neither instrument. (answer given as a fraction)</p> <p>c) A student is chosen at random, <b>calculate</b> the probability that this student plays guitar only. (answer given as a fraction)</p>	<p>3 marks</p> <p>1 mark</p> <p>1 mark</p>
<p>S8</p>	<p>Let <math>A</math> and <math>B</math> be two events such that <math>p(A) = 0.6</math>, <math>p(B) = 0.2</math> and <math>p(A \cup B) = 0.7</math></p> <p>a) <b>Calculate</b> <math>p(A \cap B)</math>.</p> <p>b) Are <math>A</math> and <math>B</math> independent? <b>Justify</b>.</p> <p>c) <b>Calculate</b> <math>p(B/A)</math>.</p>	<p>1 mark</p> <p>2 marks</p> <p>2 marks</p>

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<p>S9</p>	<p>A player draws a card from a pack of 52 cards.</p>  <p>Let <math>X</math> be the random variable that will count points as follows :</p> <ul style="list-style-type: none"> <li>- cards with face value 2 to 9 give 1 point</li> <li>- cards with face value 10 give 5 points</li> <li>- jacks, queens, kings give 10 points</li> <li>- aces give 20 points</li> </ul> <p>a) <b>Give</b> the probability distribution of <math>X</math>.</p> <p>b) <b>Calculate</b> the probability that the player gets at least 10 points (answer given as a fraction).</p> <p>c) <b>Calculate</b> the expected value of <math>X</math> (answer given as a fraction).</p>	<p>2 marks</p> <p>1 mark</p> <p>2 marks</p>
<p>S10</p>	<p>Ginkgo biloba is a tree species frequently planted in urban areas as it is resistant to pollution and easy to maintain. However, it happens that some trees produce very bad smelling fruits. A town is willing to plant 30 ginkgos in a street. They contact a tree grower who states only 10% of his trees will have smelly fruits.</p> <p>We assume that the random variable <math>X</math> that counts the number of smelly trees follows a binomial distribution.</p> <p>a) <b>Give</b> the parameters of that binomial distribution.</p> <p>b) <b>Calculate</b> the expected number of trees with smelly fruits.</p> <p>c) <b>Write down</b> the formula that would calculate the probability that none of the trees would have smelly fruits.</p>	<p>1 mark</p> <p>2 marks</p> <p>2 marks</p>

Annex page to be handed in

Question S2

