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| **MATHEMATICS 3 PERIODS****PART B** |

**DATE:** January, Monday the 29th, 2024

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| **TIME ALLOWED FOR THE EXAM:**2 hours (120 minutes)**AUTHORISED MATERIAL:**● Examination with technological tool: Approved calculator● Pencil for the graphs● Formula booklet**PARTICULAR REMARKS:** |  |

● Answers must be supported by explanations.

● Full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.

● When the answer provided is not the correct one, some marks can be awarded if it is evident that an appropriate method and/or a correct approach has been used.

**NUMBER OF EXAM DOCUMENTS: 2**

**EXAM DOCUMENTS:**

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| **EXAM PAPER** | **YES ⊠ NO** [ ]  |
| **ANSWER BOOKLET** | **YES** [ ]  **NO ⊠** |
| **FORMULA BOOKLET** | **YES ⊠ NO** [ ]  |
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**NUMBER OF PAGES OF THE EXAM PAPER: 8**

*REMINDER: NO ANSWERS TO BE WRITTEN ON THE EXAM PAPER*

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**NAME OF PUPIL:** …………………………………

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| PART B |
| QUESTION B1 | Page 1/3 | Marks |
| *In this question, parts 1, 2 and 3 are independent.* |  |
| **Part 1.**Sports watches are wristwatches that can be used during sporting activities. A lot of people use those watches.The so-called *Sporty* sports watch is particularly popular. The probability that a random person with a sports watch has the watch *Sporty* is 60 %.We are looking at a sample of 500 people with sports watches. The random variable $X$ gives the number of people in this sample that have the sports watch *Sporty*. | Ein Bild, das Text, Uhr, Schrift, Zahl enthält.  Automatisch generierte Beschreibung |  |
|  |  |
| a) | **Explain** why $X$ can be modelled by a binomial law and **give** its parameters. | 2 marks |
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| b) | **Calculate** the probability that at least 300 people in this sample have the sports watch *Sporty*. **Round** to 2 decimal places. | 2 marks |
|  |  |  |
| c) | **Determine** the expected number of people in this sample with the sports watch *Sporty*. | 2 marks |
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| d) | **Calculate** the standard deviation of $X$. **Round** to 3 decimal places. **Interpret** it in the given context. | 2 marks |

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| PART B |
| QUESTION B1 | Page 2/3 | Marks |
| **Part 2.**The sports watch *Sporty* can give the effort during a run very accurately if the person gives his or her weight.A woman with a weight of 60 kg is running uphill for 30 minutes. Therefore, her effort level is not steady. Her running power can be modelled by the following function:$P\left(t\right)=−0.05t^{2}+3t+66$, with $0\leq t\leq 30$where $t$ is in minutes and $P\left(t\right)$ in kJ/min (kilojoules per minute). |  |
|  |  |
| e) | **Calculate** at which power the woman is running when she starts running, and 15 minutes after she started. | 3 marks |
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| f) | **Draw** the graph of the function $P$ in the given definition set. | 3 marks |
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| g) | **Determine** at what time the woman’s running power is 106 kJ/min. | 3 marks |

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| PART B |
| QUESTION B1 | Page 3/3 | Marks |
| **Part 3.**A lot of people are using the internet to buy their sports watch *Sporty*, and ask for a delivery at a shop called “RunAway”.We know that 80% of the time the *Sporty* arrives on time (in a few days), 15% of the time it arrives late (it takes some weeks to arrive) and the rest of the times it doesn’t arrive at all.We also know that when the *Sporty* arrives on time, the probability that people like the shop “RunAway” is 0.9; when it arrives late, the probability that people like it is 0.3; and if it doesn’t arrive at all the probability that people like the shop is 0.1.We randomly select a user who ordered a *Sporty* watch online and asked for delivery in this shop. |  |
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| h) | **Sketch** a tree diagram of the situation above. | 3 marks |
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| i) | **Compute** the probability that this user likes the shop “RunAway”. | 2 marks |
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| j) | If we know that this person liked the shop, **give** the probability that the *Sporty* that was ordered arrived on time. | 3 marks |

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| PART B |
| QUESTION B2 | Page 1/3 | Marks |
| *In this question, parts 1 and 2 are independent.***Part 1.**A musician plays a guitar and wishes to model its shape. The main wood box can be modeled by the following equation:$$f\left(x\right)=−0.13x^{4}+1.4x^{3}−4.9x^{2}+6x$$The following picture shows the curve of $f$ (in red, plain line), together with the symmetric of this curve, with respect to the $x$-axis (in blue, dashed line). In this equation, $x$ is in decimetres, and $f\left(x\right)$ is also in decimetres. The surface between those two curves forms the wood box of this guitar. |  |
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| As can be seen on the graph, the function $f$ is in fact defined from $0$ to a value $w$, which is the other solution of the equation $f\left(x\right)=0$. |  |

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| PART B |
| QUESTION B2 | Page 2/3 | Marks |
| a) | **Determine** the value of $w$, **rounding** to 3 decimal places. **Give** the length of the wood box, in centimetres. | 2 marks |
|  |  |  |
| b) | **Determine** the maximum value of $f$, **rounding** to 3 decimal places. **Give** the width of the wood box, in centimetres. | 2 marks |
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| c) | The function $f$ has three stationary points. In question b) we have found one of them. **Give** the coordinates of the two other stationary points, **rounded** to two decimal places. | 4 marks |
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| Before a big concert, our musician wants to paint the back of the wood box in black. We hence want to know what is the area of this surface. |  |
| d) | **Determine** an approximate value of the following integral, **rounded** to 3 decimal places:$$\int\_{0}^{5.3}f\left(x\right)dx$$**Give** the area that has to be painted, in square decimetres. | 3 marks |

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| PART B |
| QUESTION B2 | Page 3/3 | Marks |
| **Part 2.**Our musician opens a webpage for his band, and is interested in the number of followers across time ($x=0$ when the webpage is created). The table below shows the number of followers for the first 20 weeks:

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $x$ = Time (weeks) | 2 | 4 | 5 | 8 | 10 | 11 | 12 | 13 | 16 | 18 |
| $y$ = Number of followers | 275 | 240 | 180 | 300 | 380 | 350 | 250 | 350 | 440 | 400 |

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| e) | **Draw** a scatter diagram to represent the data from the table. | 3 marks |
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| f) | **Compute** the linear correlation coefficient. **Determine** if a linear model would be appropriate for his data. **Discuss** how we could improve the linear model by combining it with another one. | 3 marks |
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| g) | **Determine** an equation in the form $y=a⋅x+b$ of the linear regression of $y$ on $x$ using this data. Round $a$ and $b$ to one decimal place.**Draw** the regression line on the same diagram as e). | 3 marks |
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| In h) and i), use the linear model $f\left(x\right)=20⋅x+190$. |  |
| h) | **Compute** when the number of followers would be over 800. | 3 marks |
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| i) | **Explain** why the model is not appropriate over many weeks. | 2 marks |

**END OF THE EXAMINATION**